In [408…

```
import numpy as np
import matplotlib.pyplot as plt
import astropy.constants as con
import astropy.units as u
from astropy.table import Table
%matplotlib inline
```

Gravitational Lensing Formula Special Case

The deflection is defined by $\tilde{\alpha}$ where M(ξ) is the mass within the distance of closest approach, ξ

$$\tilde{\alpha} = \frac{4GM(\xi)}{c^2}\frac{1}{\xi^2}$$

For the sun, $\tilde{\alpha} = \frac{4GM_\odot}{c^2}\frac{1}{R_\odot^2}$ = 1.74 arcseconds

1 arcsecond = 1/3600 degress

$$\alpha(\theta) = \frac{D_{ls}}{D_{os}}\tilde{\alpha}(\theta)$$

$$\beta(\theta) = \theta - \alpha(\theta)$$

$$\beta(\theta) = \theta - \frac{D_{ls}}{D_{os}D_{ol}}\frac{4GM}{c^2\theta}$$

If we set $\beta(\theta) = 0$ such that the source is just behind the lense, then we get,

$$\theta = \theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{ls}}{D_{os}D_{ol}}}$$

D_{ij} = distance from plane i to plane j

```
l= lens
o=observber
s= source
```

assuming that $D_{ls} \approx D_{ol}$

$$\theta = \theta_E = \sqrt{\frac{4GM}{c^2 D_{os}}}$$

I will write a function that can give us the angle for the various parameters

In [395…

```
a= 4*con.G*con.M_sun/con.c**2/con.R_sun
print('the deflection angle for the sun in arcsec is:')
a*206265
```

```
the deflection angle for the sun in arcsec is:
```

Out[395… 1.751192

In [ ]:

In [404…

```
# Eistein Angle Function theta_E
constant = 4 * con.G / con.c**2

Mass = input('Mass in Solar Masses = ') * u.solMass
D_ol = input('D_ol in parsec = ') * u.parsec

def theta_e(Mass, D_ol):

    E = np.sqrt(constant * Mass  /  D_ol) * u.rad

    return E.to(u.arcsec)
#E is in radians and 1 rad is 206265 arcseconds
print('\nFor example: the Einstein Angle of Sagittarious A* (black hole at the center o
print('\ntheta_e =',theta_e(Mass, D_ol))
```

```
Mass in Solar Masses = 4.154e6
D_ol in parsec = 26673

For example: the Einstein Angle of Sagittarious A* (black hole at the center of the milk
y way galaxy, with the given quantities above is:

theta_e = 1.1261916005486818 arcsec
```

In [405…

```
Mass = np.array([1, 1, 1, 4.154e6, 1e12 , 1e12, 1e12, 1e15, 1e15, 1e15]) * u.solMass
D_os = np.array([100, 1000, 10000, 26673, 100e6 , 1000e6, 10000e6, 1000e6, 10000e6, 100
E_angle = theta_e(Mass, D_os)
t = Table([Mass, D_os, E_angle], names=('Lens Mass M$_\odot$', 'D$_{os}$', '$\\theta_E$
t
```

Out[405… *Table length=10*

Lens Mass $M_\odot$	D_{os}	θ_E
solMass	pc	arcsec
float64	float64	float64
1.0	100.0	0.009024330045344293
1.0	1000.0	0.0028537437300378554
1.0	10000.0	0.0009024330045344293
4154000.0	26673.0	1.1261916005486818
1000000000000.0	100000000.0	9.024330045344295
1000000000000.0	1000000000.0	2.8537437300378556
1000000000000.0	10000000000.0	0.9024330045344293
1000000000000000.0	1000000000.0	90.24330045344293
1000000000000000.0	10000000000.0	28.537437300378553

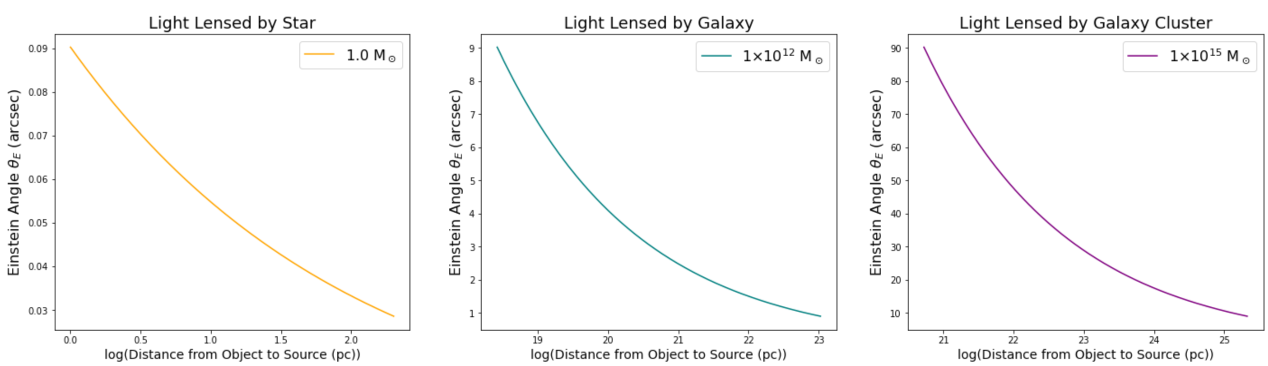
Lens Mass $M_\odot$	**D_{os}**	θ_E
1000000000000000.0	100000000000.0	9.024330045344295

In [407…

```
x1 = np.linspace(1,1,1000) * u.solMass
y1 = np.linspace(1,10, 1000) * u.parsec
E1=theta_e(x1,y1)
x2 = np.linspace(1e12,1e12,1000) * u.solMass
y2 = np.linspace(100e6,10000e6, 1000) * u.parsec
E2=theta_e(x2,y2)
x3 = np.linspace(1e15,1e15,1000) * u.solMass
y3 = np.linspace(1000e6,100000e6, 1000) * u.parsec
E3=theta_e(x3,y3)

fig, (ax1,ax2,ax3) = plt.subplots(1, 3, figsize=(24,6))

ax1.plot(np.log(y1/u.parsec), E1, color= 'orange', label = '1.0 M$_\odot$')
ax1.set_title('Light Lensed by Star',  fontsize=18)
ax1.set_ylabel('Einstein Angle $\\theta_E$ (arcsec)',  fontsize=16)
ax1.set_xlabel('log(Distance from Object to Source (pc))', fontsize=14)
ax1.legend(fontsize=16)
ax2.plot(np.log(y2/u.parsec), E2, color= 'teal', label = '1$\\times 10^{12}$ M$_\odot$'
ax2.set_title('Light Lensed by Galaxy', fontsize=18)
ax2.set_ylabel('Einstein Angle $\\theta_E$ (arcsec)', fontsize=16)
ax2.set_xlabel('log(Distance from Object to Source (pc))', fontsize=14)
ax2.legend(fontsize=16)
ax3.plot(np.log(y3/u.parsec), E3, color= 'purple', label = '1$\\times 10^{15}$ M$_\odot
ax3.set_title('Light Lensed by Galaxy Cluster', fontsize=18)
ax3.set_xlabel('log(Distance from Object to Source (pc))', fontsize=14)
ax3.set_ylabel('Einstein Angle $\\theta_E$ (arcsec)', fontsize=16)
ax3.legend(fontsize=16)
plt.show()
print('as you can see, the einstein angle is very small for Stars but large for galaxie
fig.savefig('gravitational_lensing', dpi=300)
```




```
as you can see, the einstein angle is very small for Stars but large for galaxies and ga
latic clusters
```

Magnificaton Equation

Plugging In constants, the Einstein Angle is:

$$\theta_E \approx 1.8\sqrt{\frac{M}{10^{12}M_\odot}}$$

Using $u = \frac{\beta}{\theta_E}$

$$\mu_{\pm} = \frac{u^2+2}{2u\sqrt{u^2+4}} \pm \frac{1}{2}$$

$$\mu = |\mu_{+}| + |\mu_{+}| = \frac{u^2+2}{u\sqrt{u^2+4}}$$