

The background is a vibrant, stylized space scene. It features large, flowing nebulae in shades of red, purple, and blue. Scattered throughout are various celestial bodies: a large red planet with orange and yellow patterns in the top right, a yellow and orange striped planet in the bottom left, and several smaller blue and purple planets. White stars of different sizes are also visible.

# GRAVITATIONAL LENSING

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01

# BACKGROUND INFO

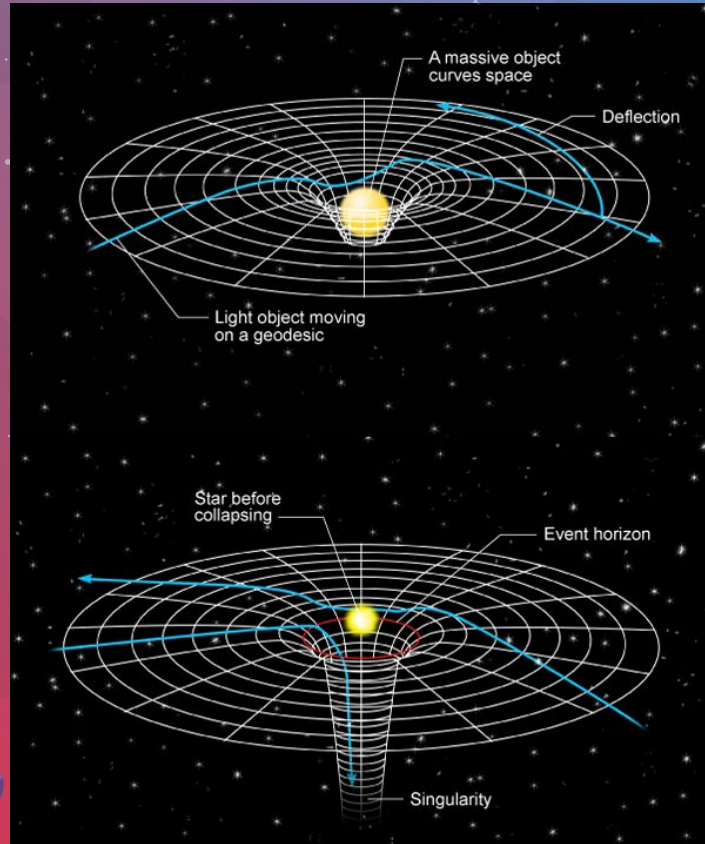


# BACKGROUND INFO

- General Relativity: mass warps space-time
  - Think of space-time as a 2D blanket
- When an object is placed on it, the blanket stretches out

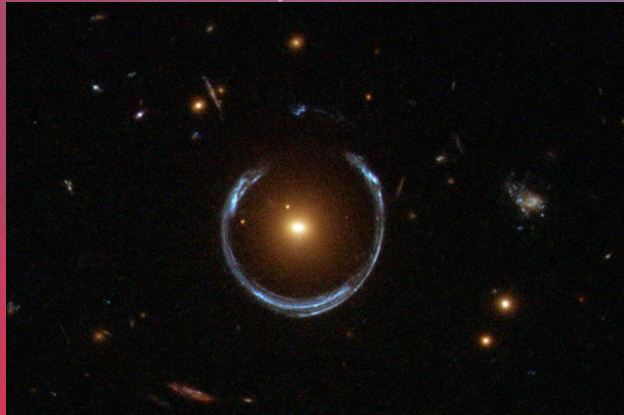


# MORE MASS = MORE STRETCHING!



# WHAT IS GRAVITATIONAL LENSING?

- The effect of the massive object curves traveling light!
- This causes objects to appear in different places than they actually are
- Can also change object's appearance entirely with enough curvature!



The background is a vibrant, stylized space scene. It features a gradient of colors from deep red on the left to light blue on the right. Scattered throughout are various celestial bodies: a planet with blue and white stripes in the top left, a planet with orange and white stripes in the top right, a planet with blue and white stripes in the bottom right, and a planet with yellow and red stripes in the bottom right. There are also several small, dark blue planets and numerous white stars of varying sizes.

02

# Theory / Relevant Formulas

# 1 Gravitational Lensing Formula Special Case

The deflection is defined by  $\tilde{\alpha}$  where  $M(\xi)$  is the mass within the distance of closest approach,  $\xi$

$$\tilde{\alpha} = \frac{4GM(\xi)}{c^2} \frac{1}{\xi^2}$$

For the sun,  $\tilde{\alpha} = \frac{4GM_{\odot}}{c^2} \frac{1}{R_{\odot}^2} = 1.74$  arcseconds

\*\*1 arcsecond = 1/3600 degree

$$\alpha(\theta) = \frac{D_{ls}}{D_{os}} \tilde{\alpha}(\theta)$$

$$\beta(\theta) = \theta - \alpha(\theta)$$

$$\beta(\theta) = \theta - \frac{D_{ls}}{D_{os}D_{ol}} \frac{4GM}{c^2\theta}$$

If we set  $\beta(\theta) = 0$  such that the source is just behind the lense, then we get,

$$\theta = \theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{ls}}{D_{os}D_{ol}}}$$

$D_{ij}$  = distance from plane i to plane j

l= lens

o=observer

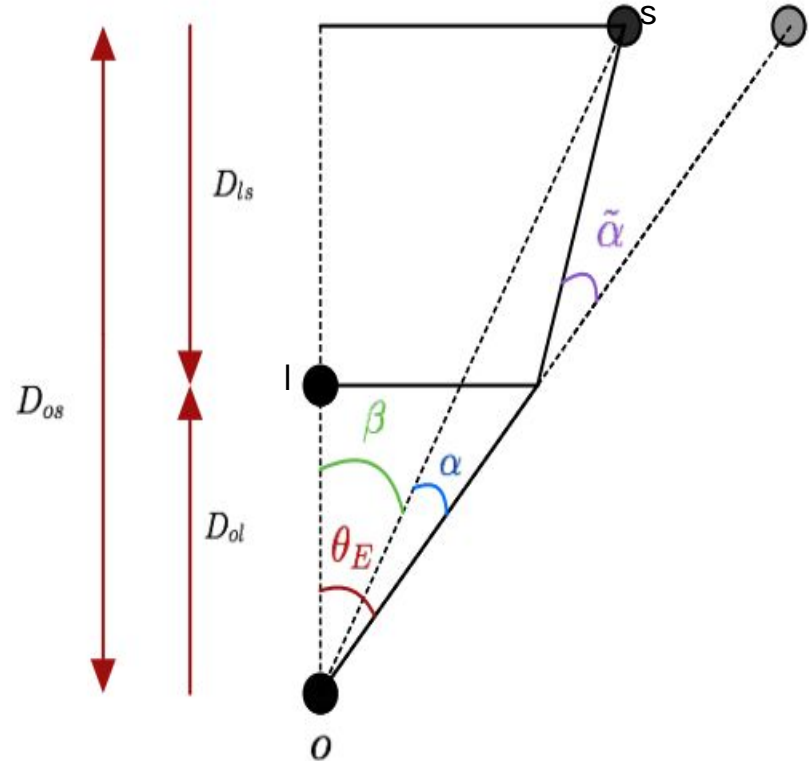
s= source

assuming that  $D_{ls} \approx D_{ol}$

$$\theta = \theta_E = \sqrt{\frac{4GM}{c^2 D_{os}}}$$

I will write a function that can give us the angle for the various parameters

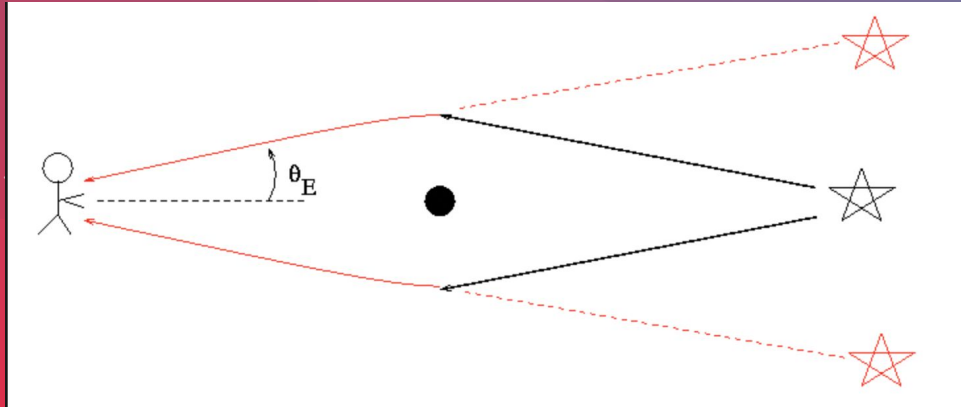
# L<sup>A</sup>T<sub>E</sub>X



# RELEVANT EQUATION

$$\theta_E = \sqrt{\frac{4GM}{Dc^2}}$$

- As mass increases, the angle of deflection increases
- Larger distance creates a smaller angle

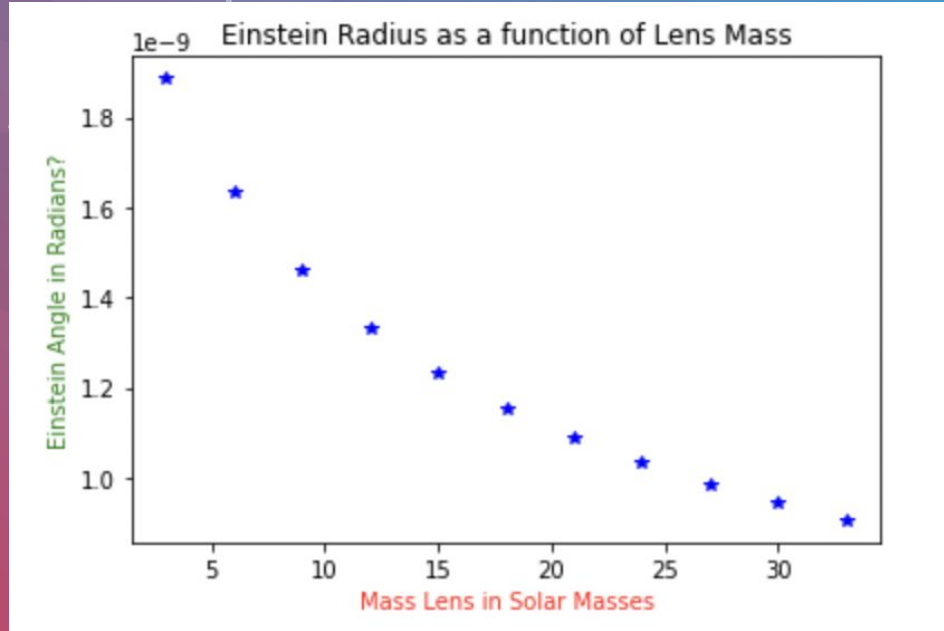


The background is a vibrant, abstract representation of outer space. It features a gradient from deep red on the left to bright blue on the right. Large, flowing, organic shapes in shades of purple, pink, and blue suggest the structure of galaxies or nebulae. Scattered throughout are numerous small white dots representing stars, along with several four-pointed white sparkles. A few stylized planets are visible: a small red one in the upper left, a blue one in the lower left, a purple one in the upper right, and a larger one with vertical blue and white stripes in the middle right. In the top right corner, a bright yellow and orange sun or star is partially visible.

03

# METHODS/TECHNIQUES

# INITIAL PLOT OF ANGLE VS. MASS



# Einstein Angle Python Function

```
# Einstein Angle Function theta_E
constant = 4 * con.G / con.c**2

Mass = input('Mass in Solar Masses = ') * u.solMass
D_ol = input('D_ol in parsec = ') * u.parsec

def theta_e(Mass, D_ol):

    E = np.sqrt(constant * Mass / D_ol) * u.rad

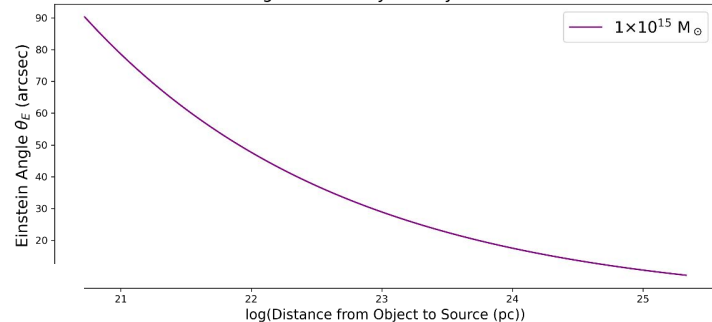
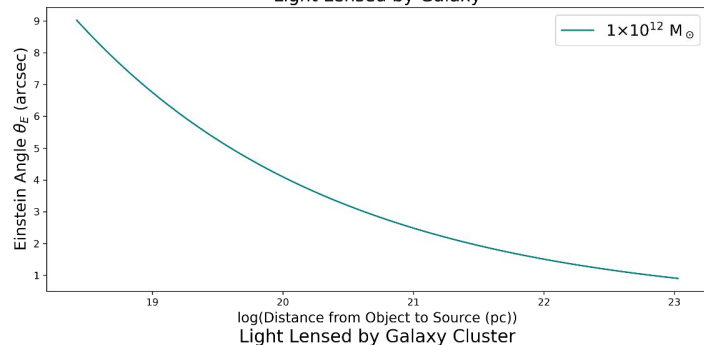
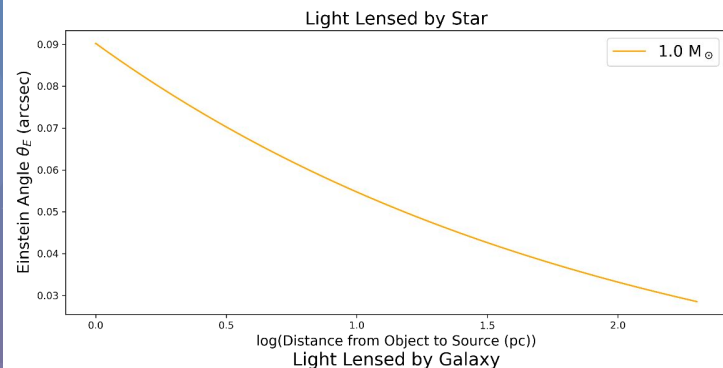
    return E.to(u.arcsec)

#E is in radians and 1 rad is 206265 arcseconds
print('\nthe Einstein Angle of Sagittarius A* (black hole at the center of the milky way galaxy)')
print('\ntheta_e = ',theta_e(Mass, D_ol))
```

```
Mass in Solar Masses = 4.154e6
D_ol in parsec = 26673
```

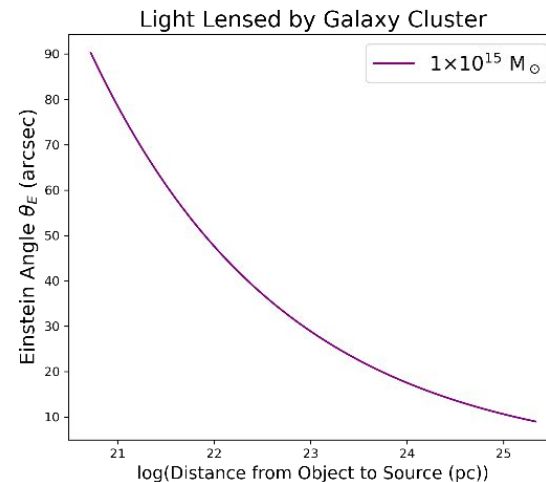
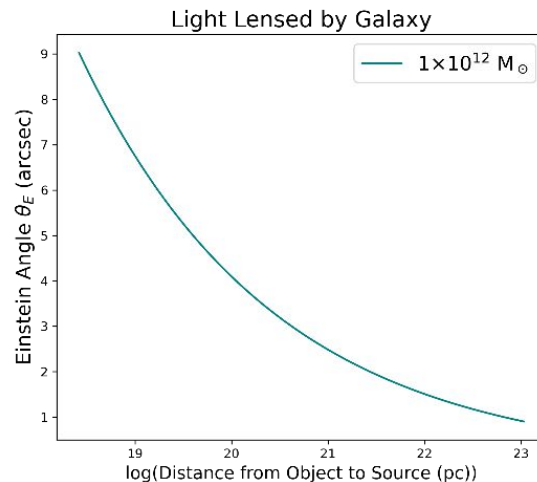
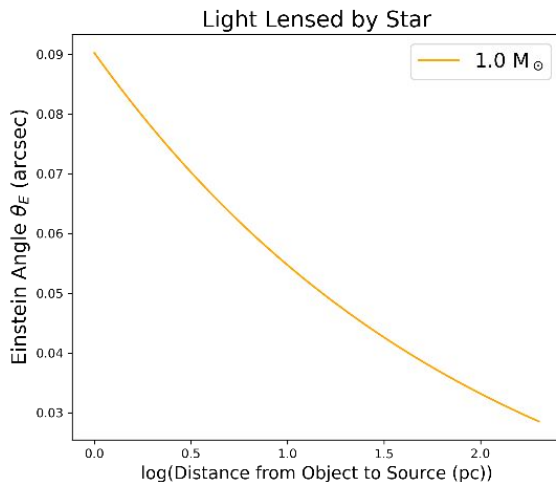
the Einstein Angle of Sagittarius A\* (black hole at the center of the milky way galaxy)

```
theta_e = 1.1261916005486818 arcsec
```



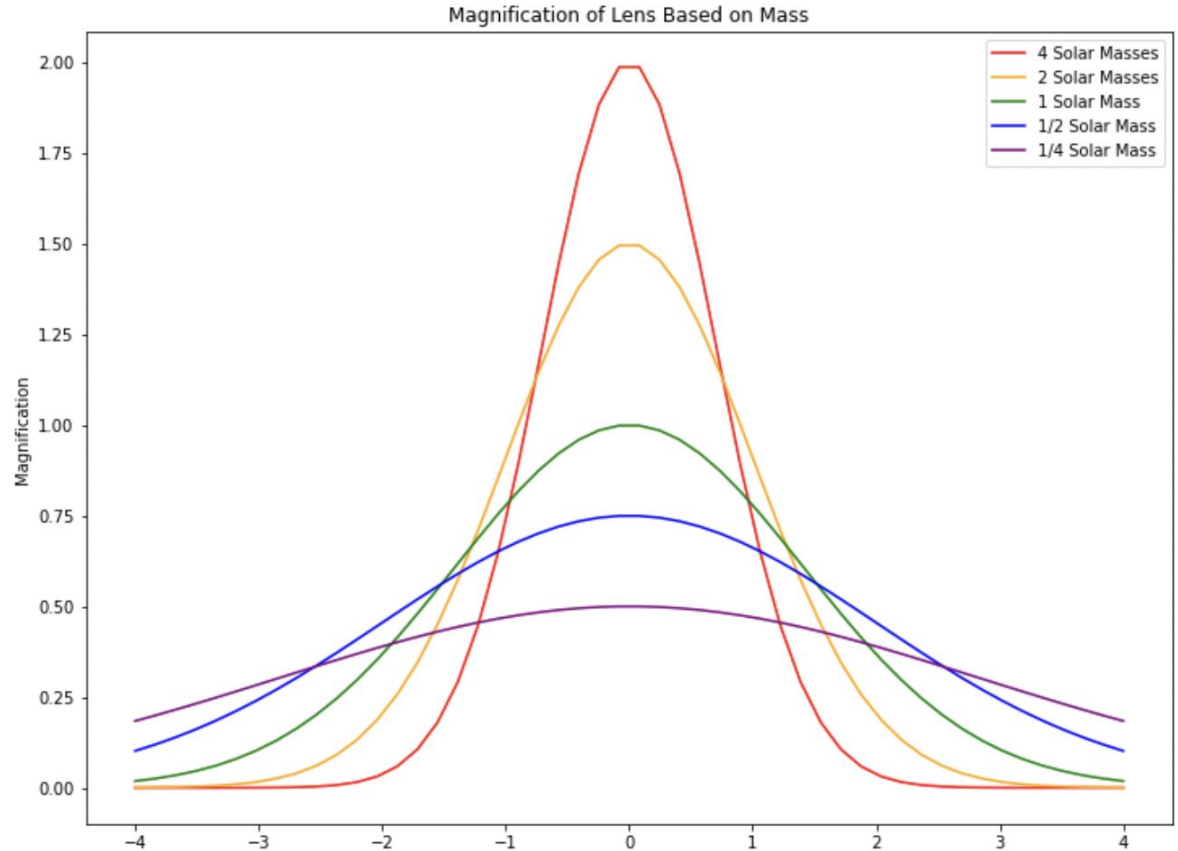
Need a Massive Object, some stars just do not deflect light enough, but blackholes, galaxies, galaxy clusters do!

| Lens Mass $M_{\odot}$ | $D_{os}$        | $\theta_E$            |
|-----------------------|-----------------|-----------------------|
| solMass               | pc              | arcsec                |
| float64               | float64         | float64               |
| 1.0                   | 100.0           | 0.009024330045344293  |
| 1.0                   | 1000.0          | 0.0028537437300378554 |
| 1.0                   | 10000.0         | 0.0009024330045344293 |
| 4154000.0             | 26673.0         | 1.1261916005486818    |
| 1000000000000.0       | 100000000.0     | 9.024330045344295     |
| 1000000000000.0       | 1000000000.0    | 2.8537437300378556    |
| 1000000000000.0       | 10000000000.0   | 0.9024330045344293    |
| 1000000000000000.0    | 10000000000.0   | 90.24330045344293     |
| 1000000000000000.0    | 100000000000.0  | 28.537437300378553    |
| 1000000000000000.0    | 1000000000000.0 | 9.024330045344295     |



If the black hole/star is more massive, the magnification increases!

$$\mu = \frac{u^2 + 2}{u\sqrt{u^2 + 2}}$$



# RESOURCES

## LECTURES

- Astro 7B Lecture 25 (Spring 2020)  
taught by Professor Mariska Kriek

## Academic Papers

Abdo, A. "Department of Physics and  
Astronomy Michigan State  
University East Lansing, Michigan  
48823."